Utilizing Mathematical Modeling Techniques to Optimize Athletic Footwear for Sports Performance

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Mathematical modeling is a technique used to predict specific outcomes, find optimization of structure and behavior or to simplify reality. An effective model must be designed for a specific function, and or to answer a question or hypothesis (Alexander, 2003). In the field of biomechanics, many types of models have been used to mimic walking mechanics, jumping mechanics and predict ground reaction forces. Specifically applied to running mechanics, many models take the simplified approach of assuming complete rigidity within each linked segment, such as treating the feet as one unit instead of taking into account all the intertarsal, tarsometatarsal, metatarsophalangeal, and interphalangeal joints. Because there are fewer input variables, the uncertainty margin in the prediction is smaller but the inaccuracy is inherently larger. As the number of input variables increases, such as taking into account more lower limb joints and degrees of freedom, the total uncertainty margin may increase, but the accuracy should increase.

There are three primary models I will be analyzing on their effective application to actual human movement and how they can be utilized to optimize athletic shoe design for long distance running performance of elite athletes. The first model is relatively new and consists of a tri-dimensional foot contact model that is able to detect contact between foot and ground which is dependent upon a set of spheres located under the planter surface (Figure 1). When contact is verified appropriate physical laws are applied to each to simulate interaction (Silva and Flores, 2010). This allows a much more detailed representation of the intricacies of the foot biomechanics during the gait cycle compared to older models. The second model was created by Zadpoor et al. (2007) and utilizes a mass-spring system to simulate body weight onto a foot/ground interface. A large number of parameters are discussed and utilized such as shoe stiffness and damping. Utilizing these parameters, optimization can occur to illuminate which features are most important to the foot/ground interaction. The last model is the running economy model. Running economy is a reflection of the amount of inspired oxygen required to maintain a given velocity and is considered a determining factor for running performance (Sinclair, 2015). Although not considered a pure biomechanical model, biomechanical factors play into the total value of running economy and can be a predictive value for energy consumption. For distance running, many factors play into variance in distance running. One of which is how efficiently the runner moves (Foster & Lucia, 2012). More detail of running economy comes from physiological calculations and data collection, this will not be explored in this paper, but running economy as a model for endurance performance will be used to evaluate energetic expenditure of biomechanical optimization. For the first two model, the exact numerical output data will not be compared to empirical data. This is due to the fact that it is highly dependent on the specified values such as mass and touchdown velocity that I cannot equate to the empirical studies. Instead output graphs will be analyzed and compared because the general shape and relationship holds independent of parameter values.

In the walking gait cycle, the main stages that include an interaction with the ground are Initial contact, loading response, terminal stance and pre swing. These stages include both single and double support where the foot goes from heel contact to toe lift off (assuming a normal rear foot striker gait pattern). When the walking velocity increases, it reaches a point when the distinct characteristics of the movement change and running begins. During human locomotion, the body is exposed to repetitive impact force stress. When the human body makes contact with the ground, impact, or ground reaction, forces are produced. The human body has many shock absorbers, usually classified as passive and active shock absorbers. Passive shock absorbers include bone, cartilage and synovial fluid while active shock absorbers include joint positioning and muscle activity. Prolonged exposure to high impact forces, also known as ground reaction forces, can cause fatigue of active shock absorbers and stress on passive shock absorbers. This can lead to running mechanics breakdown, decreased performance and injury (Christina, White, Gilchrist, 2001)(Malisoux et al. 2017).

Utilizing the details of the foot to ground interaction and the data of frequency, time and direction of interaction, can provide the framework needed to create a running shoe that is able to store potential energy by compression of the midsole and release this energy back to the foot as the next gait cycle occurs. Although the typical return is approximately less than 1% of the energy needed for each sequential step (Nigg, MacIntosh, 2000), over a long distance event, such as a marathon, that could be between 20,000 – 30,000 steps (Nigg, MacIntosh, 2000) This energy return could make a significant impact on the total energy expenditure and lead to improved performance.

Statisticians and sports scientists have long attempted to predict future athletic achievements based on previous records. In 1998, Liu and Schutz predicted marathon times would not reach 2:02:39 until the year 2050. While Weiss et al. concluded a sub-2-hour marathon is unlikely to happen before the year 2100. This 2-hour limit has become an iconic barrier for the world of sport prompting many to ask, “Is it possible for humans to get faster?” and if so, how do we do it. My research will tie together the field of math modeling and gait biomechanics in the hopes to show an optimal process for future sports performance application, specifically in improving distance running performance. This will also be discussed with current breakthroughs in the field including the Nike Vaporfly 4%, the 4% moniker was given because of the proposed and tested energetic savings of ~4%, and the first sub 2-hour marathon run by Kenyan long-distance runner Eliud Kipchoge.

Gait Mechanics

The gait cycle consists of two distinct phases, the stance phase and the swing phase. What is distinctly different from walking and running is that running gait does not include a double stance phase and also incorporates a float phase. Within the stance phase are two subphases of absorption and propulsion. Absorption occurs immediately after initial contact when the foot first comes in contact with the ground. Energy from this interaction is directed up into the body and absorbed by muscles, connective tissues as well as the shoe itself. The absorption phase continues until midstance is reached. From here, the remainder of the stance phase is the subphase of propulsion when energy is redirected back into the ground to propel the body forward until the foot has totally removed from the ground, this is called the toe off, or initial swing phase. The swing phase follows the stance phase and includes two periods of double floating where neither foot is in contact with the ground and an initial and terminal swing of the swing leg.

In the stance phase, the talocrural joint allows for plantarflexion and dorsiflexion, the subtalar join allows for inversion and eversion and the metatarsophalangeal joint allows for flexion and extension. For complexities sake, we will assume that the intertarsal joints and tarsometatarsal joints are stable and no motion will occur due to the presence of ligaments and each joint being classified as a planar nonaxial/nonplanar joint. These three sets of joints, and the muscles that have actions around them, work together during the stance phase to control for and optimize pressure, force, ground contact time, and therefore, impulse with the ground. (Dugan & Bhat, 2005). A key factor that is distinct to running gait biomechanics is that there is an increased ground reaction force compared to walking. This increase can put significantly more stress on not only the lower limb but entire body. Being able to decrease this stress and use this ground reaction force to store potential energy is one of the main goals of performance footwear.

Shoe Design Background

In running, shoes are designed to improve performance. A running shoe is made up of the following four components:1) upper, 2) midsole, 3) last, and 4) the outsole (Asplund & Brown, 2005). The upper is designed for comfort and to be lightweight to decrease the total weight of the shoe. The midsole is one of the most important parts of the running shoe due to its role as a cushion, stabilizer and motion controller. The last is a descriptor for the curvature of the shoe and the outsole is the part of the shoe that makes contact with the ground (Asplund & Brown, 2005). The midsole will be the feature that is of specific focus due to its properties directly contributing to absorption and propulsion effects described in the previous paragraphs. It has been theorized that properties of damping and stiffness in the midsole heavily impact biomechanical load and energy return. In a study by Cigoja et al. (2019) the properties of midsole bending stiffness were investigated on the basis of redistributing lower limb joint work. They tested 13 male recreational runners and found that positive work in the lower limb had been redistributed from the knee to the metatarsophalangeal (MTP) joint when a carbon fiber plate was inserted along the midsole of the control shoe. This was achieved by inserting the plate along the full length of the shoe and placed directly above the midsole. For comfortability, the liner and sole were placed above the plate. There was a significantly larger MTP moment due to increased vGRF at peak positive power, this was followed by earlier onset MTP flexion velocity. Concluding that the inclusion of the carbon fiber plate could be indicative of greater potential energy storage and return than the control shoe.

Mathematical Model Analysis

How the mathematical models come into the design process are with the goal to simulate each step of the running gait cycle and predict certain outcomes. Utilizing models allows control of parameters of the human body and the shoe, this can include values such as lower limb mass, spring constant of muscle tendon and dampening coefficient of the shoe cushioning (Zadpoor et al. 2007). Because The ultimate goal of this design is to reduce the peak ground reaction forces on the body, models that focused on simulating this interaction were highlighted. The first is one designed by Zadpoor et al. that investigates impact forces during running. Their model was built upon an older model developed by Liu and Nigg (2000). They created a four degree of freedom mass-damper system (Figure 2) to represent the upper and lower body masses with linear springs and dampers connecting each mass. This allowed simulation of entire body weight being absorbed and propelled in the ground reaction model. Many studies had been performed on the original Liu and Nigg model since 2000, creating optimized values for mass distribution and spring/damping coefficient values. To further improve the effectiveness, Zadpoor et al. minimized the differences between simulated and experimental results by utilizing a MATLAB code to solve a nonlinear optimization. Through simulated results, they found that when a shoe has a lower damping coefficient, being described as a "hard" shoe, it will have a significantly higher peak ground reaction force than a shoe with a higher dampening coefficient, described as a "soft" shoe (Figure 3). The results were valid only for a small time after impact because active muscle contraction is not accounted for, only absorptive dampening. Thus it will explain the impact forces during the initial contact marking the start of the absorption phase of the gait cycle. This is where peak stress occurs on the body due to the compression of joints and eccentric stretching of muscles and tendons thus learning how to minimize the effects can be key to decreasing fatigue and injury and increasing performance. Results were run for many varying values of each individual segment mass and mass ratios (Figure 4). All results held the same shape with varying magnitudes of peaks as expected. Zadpoor et al.'s model and simulation were able to incorporate a variety of parameters and produce experimentally validated results.

The foot-ground interaction is so intricate, one model cannot describe the entirety of what is occurring without sacrificing some accuracy. Silva and Flores were able to create a multi-component model of the foot by making the foot a two-segment model of the plantar surface and the toes (Figure 5). This model and simulation have allowed for the placement of nine spheres underneath the foot (Figure 1). Based on their configuration the model can detect when each element is interacting with each other. This allows for the calculation of predicted ground reaction forces in all three planes of motion as well as the location of center of pressure. Through a forward dynamic analysis, ground reaction forces were calculated. It was found that the results were supported by results from literature and experimental results in both the vertical (Figure 6) (Figure 7) and anterior/posterior directions (Figure 8) (Figure 9)(Silva & Flores, 2010). The vertical ground reaction had a bimodal peak distribution representing the first initial GRF peak during the initial absorption phase and then a secondary peak that represents the push off effect during the propulsion phase of running gait (Figure 6). The final toe off stage is simulated using the metatarsophalangeal joint as a revolute (having one degree of freedom)(Figure 5) joint with a torsional spring and damper. Meaning that it can rotate along one axis and that during dorsiflexion of the phalanges in the foot during the end of stance phase stores potential energy then releases it during toe off when the phalanges start to plantar flex.

These models act as a guide for researchers to fully understand the foot/ground interface and instead of investing large sums of money and time into conducting empirical research for different scenarios of parameter combinations, they can all be tested simultaneously and processed through the models. Do these models still need validation from empirical research? Yes of course but utilizing technology such as these can streamline the process. Applying this to increasing sports performance, these models have highlighted key factors such as decreasing peak impact GRF at high velocities by increasing the damping coefficient of the shoe. Graphs produced from the models match those from collected from empirical studies testing vertical ground reaction forces reaction (Figure 3) (Figure 10)(Kluitenberg et al. 2012) for the first initial impact loading where the model is valid. What the second model introduces is the torsional spring of the metatarsophalangeal joint and how that helps the secondary propulsion GRF peak. How this occurs without shoes, is the foot acts as a spring through the arches and the MTP complex (Stearne et al. 2016). There are two known mechanisms that contribute to this effect, the windlass and the arch spring mechanism (Figure 11) (Welte, et al. 2018). As the arch is compressed, during the absorption phase, passive-elastic energy is stored in the arch structures, when GRF increases, and released during the propulsion phase (Kelly, et al. 2016). This allows it to behave in a spring like manner, storing mechanical energy when compressed and releasing that energy when uncompressed. When paired with a stiff longitudinal plate in the shoe, elastic energy storage can increase. This is also a concept that was investigated by Crigoja et al. (2019) when the insertion of a carbon fiber plate was inserted to the midsole of the shoe. What is occurring here is that during each step the plate is being bent and storing a certain amount of (potential) energy due to the carbon fibers elastic properties. When the propulsion phase begins, extension of the metatarsophalangeal joint occurs thus bending the shoe and the carbon fiber. This is why Crigoja et al. noticed such an earlier onset of MTP flexion velocity because the stored energy was paired with intrinsic muscle torque production to accelerate the joint faster.

Thus, the objective of footwear design for running performance should be to optimize the relationship between the cushioning/damping and the innate stiffness of the sole.

Application to Sports Performance

When these two parameters (dampening and longitudinal bending stiffness) are paired with an already established value of shoe mass, full energetic cost and return can be analyzed. The value of shoe mass has been studied for a long time and it is conclusive that heavier shoes put a greater energy demand on running by about 1% for every 100g of weight (Frederick, 1984). This is due to the increased inertial value of leg swing during the swing phase of the gait cycle. In a comparison study, Hoogkamer et al. (2017), highlight studies that have found that shoes with midsoles that are more compliant and resilient, meaning that they are softer and have the ability to return stored energy and not lose original shape, can reduce the energetic cost of running by ~1% (Worobets et al. 2014). It also highlights how the foot acts as a lever and that the longitudinal bending stiffness of the shoe can reduce energetic cost of running by ~1% by changing the leverage of the metatarsophalangeal joint. In the Hoogkamer et al. study they tested a protype shoe that incorporated both highly compliant and resilient cushioning with a stiff plate (Figure 12) against established marathon shoes. They found that the prototype shoe lowered the energetic cost of running ~4% on average when compared to the other control shoes in the trials. This is a massive improvement on energetic savings for running. Now how much of a running improvement could be predicted form this energetic saving? In a different study by Hoogkamer et al. (2016), he found that running economy increase percentage is proportionally to 3000m time performance, i.e if running economy improves 4% then the 3000m time would improve by 4%. This suggests that with an improved shoe design, elite level runners could improve their times by the given energy conversation increase.

Before 2018, the 2-hour marathon barrier seemed to be a mark that humans could never cross. The world record stood at 2:02:57, to improve to a time of 1:59:59 would require an increase in speed of 2.5% (Hoogkamer, 2017). This increase in speed at already peak human performance seemed impossible, but according to Hookgamers's prediction, if a shoe whose tested energetic savings was ~4% then the time improvements should also be around 4%, thus if the time in control shoes was 2:02:57, with Hoogkamer's prediction we would expect to see a time of around 1:58:00. In 2018, Nike released the Vapormax 4% marathon running shoe that had both a highly compliant and resilient cushioning system, a carbon fiber plate inserted in the sole for increased longitudinal bending stiffness and was extremely light weight. The 4% signified the improved energy saving the shoe could provide. The shoe was worn by elite marathon runner Eliud Kipchoge when he improved his marathon time from 2:01:39 to 1:59:40, which represents an improvement of ~2%. This improvement does not entirely match up to Hookgamer's proportionality prediction, that is most likely due to the fact that air resistance was not considered and does effect high velocity running (To break the record, Kipchoge had to run at an average pace of 13.3 mph over the entire 26.2 miles). This was not just specific to male marathon runners. With the same shoes, Brigit Kosgei was able to break the women's marathon world record of 2:15:25 by a minute and twenty-one seconds by posting a time of 2:14:21 at the Chicago marathon a day after Kipchoge, Kosgei was also wearing the Nike Vaporfly 4% shoes. These immediate and groundbreaking results in the sports world show that not only did the models predict that factors such as vertical ground reaction force, damping/compliance of shoe material and the importance of bending stiffness around the metatarsophalangeal joint, they also had accurate prediction that were able to guide researchers to study these important factors because of the profound effect they had on energy expenditure.

Mathematical modeling is an extremely useful tool in advanced scientific fields such as biomechanics where studies and experiments can become extremely complex, time consuming and expensive due to the equipment and testing procedures. It allows for simulation and prediction of many experiments or trials simultaneously with full control over parameters such as shoe stiffness, body mass, shoe damping, ground stiffness, etc. The models created by Zadpoor et al. and Silva & Flores, are fully functioning optimizable models that incorporate intricacies of gait biomechanics by analyzing the kinematics and kinetics of the foot to ground interface. Specifically, in the Zadpoor et al. model, many different results were able to be produced based upon variation of parameters to see which had the greatest effect on minimizing ground reaction forces. It's findings of shoe damping coefficient being the largest contributor to minimizing was an already theorized relationship. However, where this model can be so useful is that it can show how well each damping coefficient value does given a fixed set of external parameters. This is shown to be an effective way of guiding the course of designing athletic wear due to Hoogkamer's studies involving energy expenditure based upon a prototype shoe with improvements to both midsole compliance and longitudinal bending stiffness showed a significantly decreased energy expenditure at the same velocity compared to other marathon shoes. This not only applied itself well to sports performance, it became a breakthrough of human achievement when marathon runners broke long standing records and milestones while wearing these shoes. Utilizing unique modeling techniques and simulations, it can help us answer the question "How can I run faster?" with stunning results.

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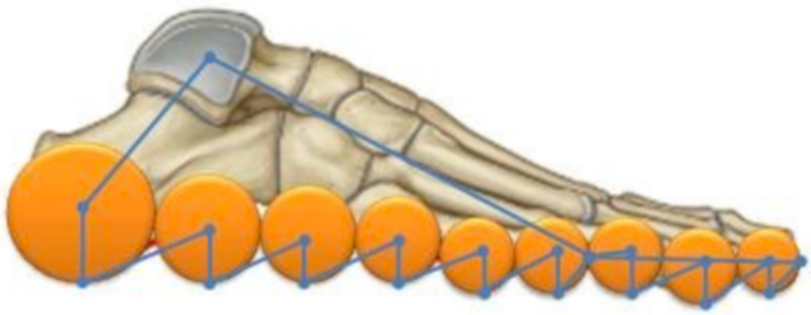
Appendum:

Figure 1:

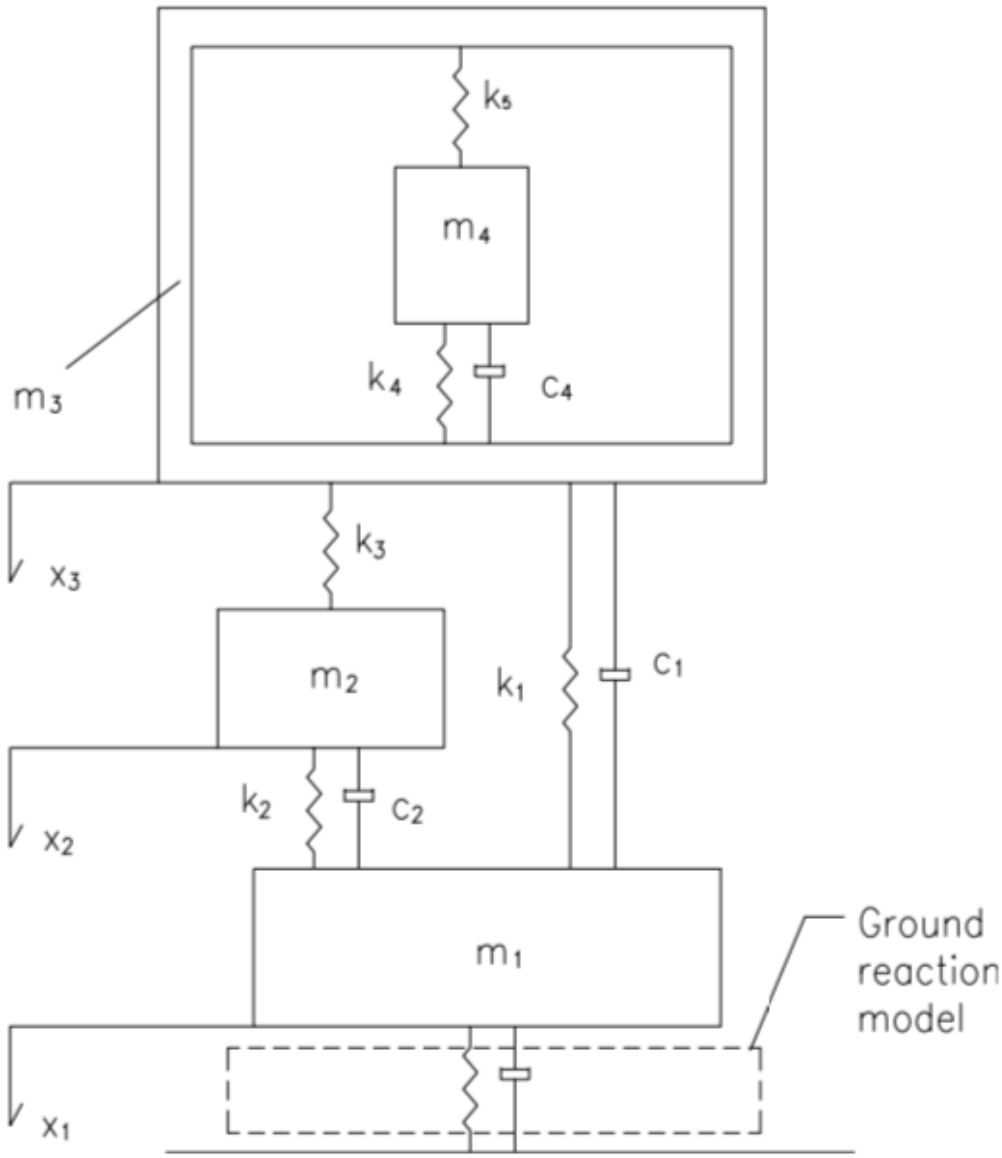
Sphere placement on the foot model



Note: Where spheres are placed along plantar surface in model to detect impact forces. Adapted from “GROUND FOOT INTERACTION IN HUMAN GAIT: MODELLING AND SIMULATION” by Moreira, P., Silva, M. T., & Flores, P. (2010), *7th EUROMECH Solid Mechanics*.

Figure 2:

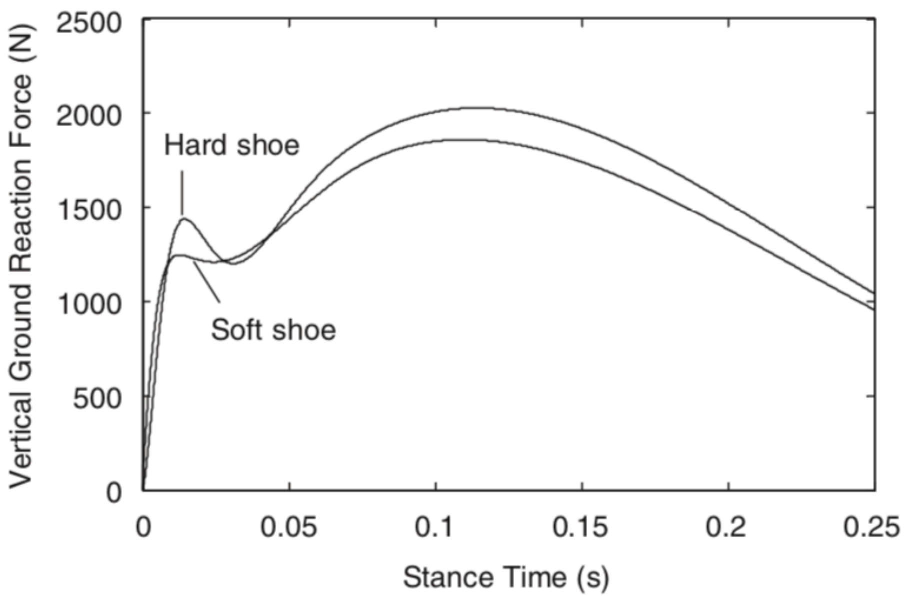
Schematic for mass-damper model



Note: Mass-damper model schematic, each m represents mass of each rigid segment, k represents spring coefficient c represents dampening coefficient, x represents vertical displacement. Adapted from “A model-based parametric study of impact force during running” by Zadpoor, A. A., Nikooyan, A. A., & Arshi, A. R. (2007), *Journal of Biomechanics*, *40*(9), 2012–2021

Figure 3:

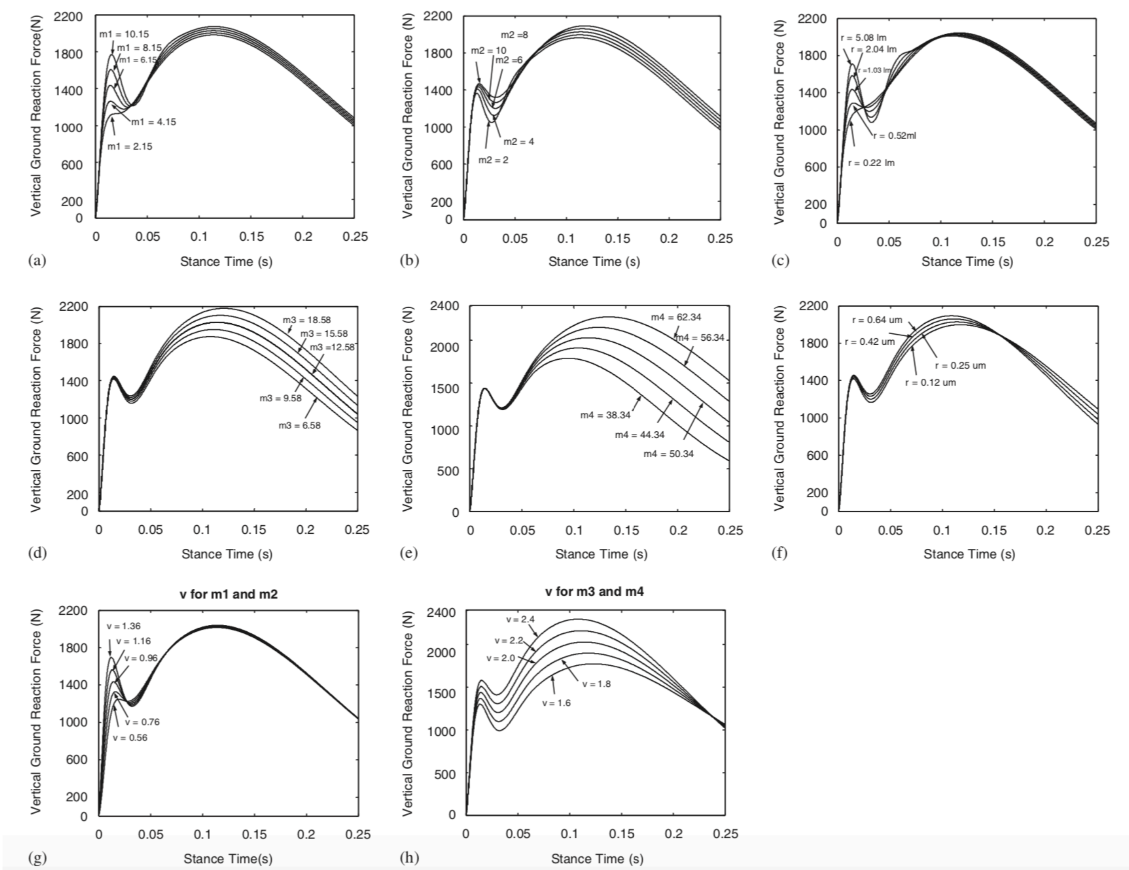
Results graph of Zadpoor Model for vGRF



Note: Results of model simulating ground reaction forces. Overlapping curves represent hard shoe and soft shoe as labeled. All other values held constant across simulations. Adapted from "A model-based parametric study of impact force during running" by Zadpoor, A. A., Nikooyan, A. A., & Arshi, A. R. (2007), *Journal of Biomechanics*, *40*(9), 2012–2021

Figure 4:

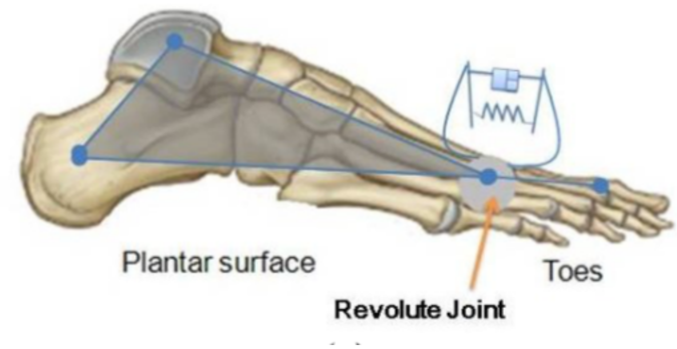
Results of Zadpoor model for different mass values of rigid segments.



Note: Result graphs for different simulation runs with different mass values. Across parameter variation, shape of curve was consistent. Adapted from “A model-based parametric study of impact force during running” by Zadpoor, A. A., Nikooyan, A. A., & Arshi, A. R. (2007), *Journal of Biomechanics*, *40*(9), 2012–2021

Figure 5:

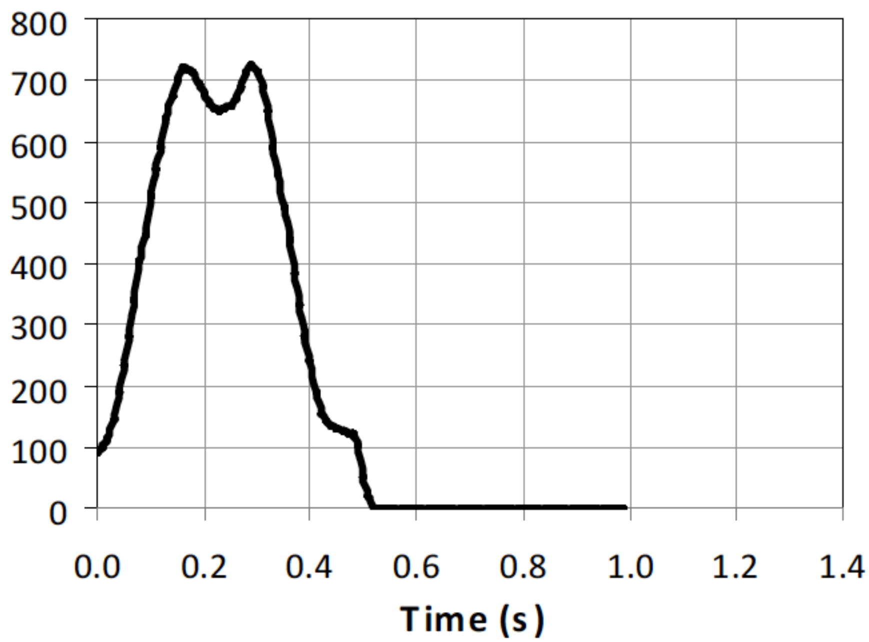
Simplification of the foot model



Note: Simplifying the foot as a two part system of the plantar surface and the toes, connected by a revolute (hinge) joint as a torsional spring. Adapted from “GROUND FOOT INTERACTION IN HUMAN GAIT: MODELLING AND SIMULATION” by Moreira, P., Silva, M. T., & Flores, P. (2010), *7th EUROMECH Solid Mechanics*.

Figure 6:

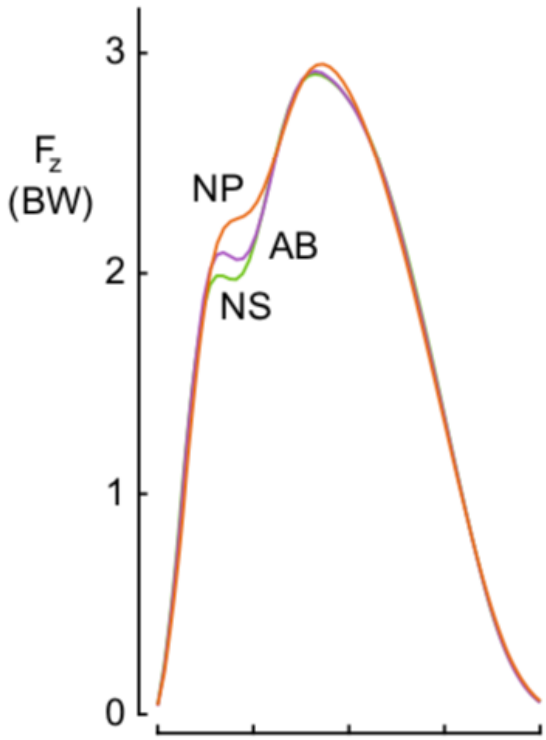
Results from Silva & Flores vGRF model



Note: Results of simulations, bimodal peak representing initial impact ground reaction force and push off ground reaction force. Adapted from "GROUND FOOT INTERACTION IN HUMAN GAIT: MODELLING AND SIMULATION" by Moreira, P., Silva, M. T., & Flores, P. (2010), *7th EUROMECH Solid Mechanics*.

Figure 7:

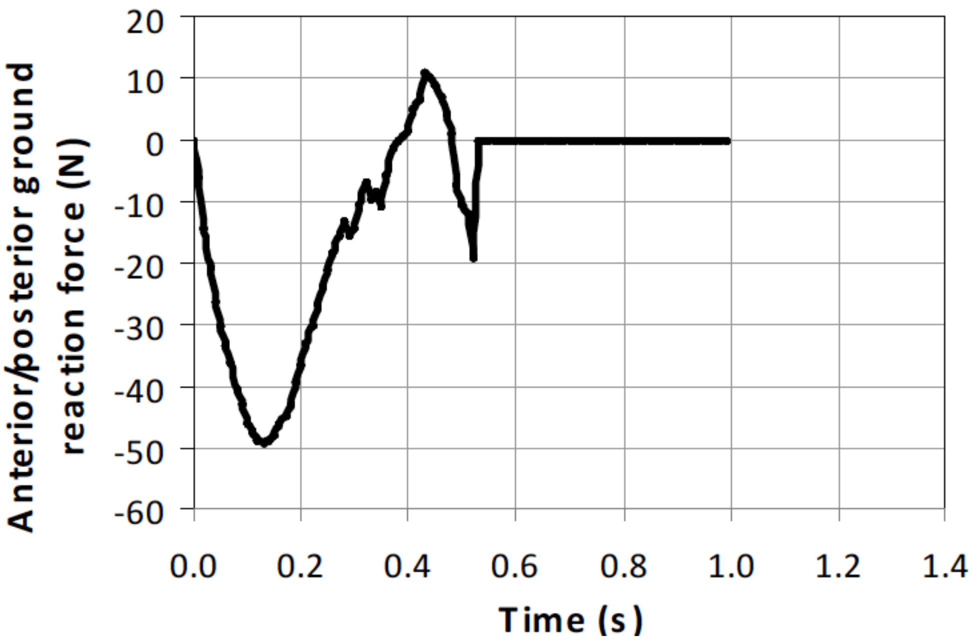
Results from Hoogkamer empirical study (vGRF)



Note: Graph showing results from Hoogkamer's empirical study of vGRF for different models of shoes. Adapted from "The Biomechanics of Competitive Male Runners in Three Marathon Racing Shoes: A Randomized Crossover Study by Hoogkamer, W., Kipp, S., & Kram, R. (2018). *Sports Medicine*, *49*(1), 133–143.

Figure 8:

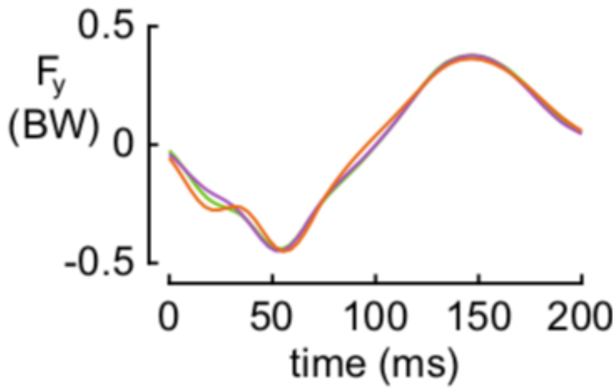
Results from Silva & Flores anterior/posterior GRF model



Note: Results of simulations, showing initial negative then positive GRF in the anterior/posterior plane. Adapted from "GROUND FOOT INTERACTION IN HUMAN GAIT: MODELLING AND SIMULATION" by Moreira, P., Silva, M. T., & Flores, P. (2010), *7th EUROMECH Solid Mechanics*.

Figure 9:

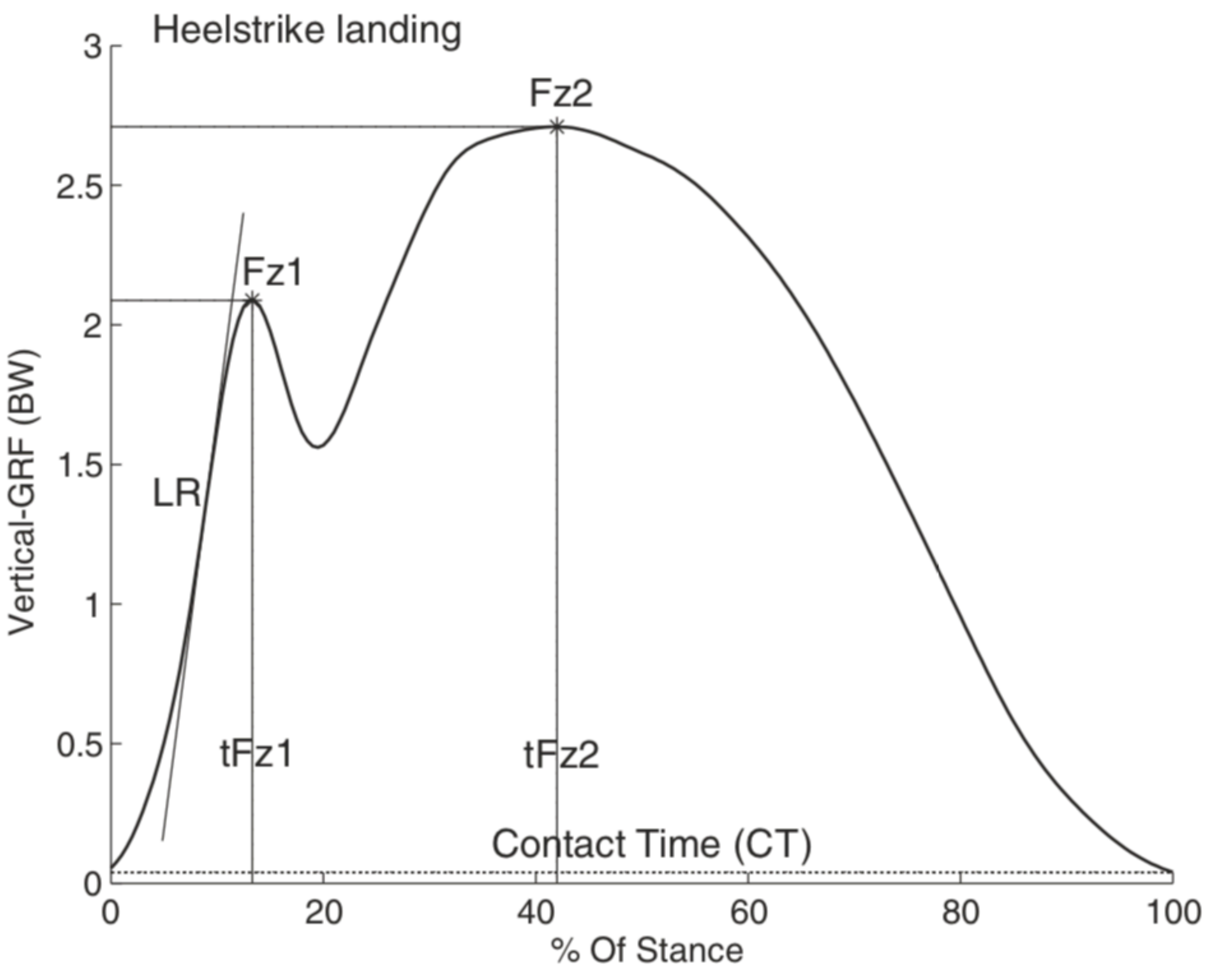
Results from Hoogkamer empirical study (anterior/posterior GRF)



Note: Graph showing results from Hoogkamer's empirical study of anterior/posterior GRF for different models of shoes. Adapted from "The Biomechanics of Competitive Male Runners in Three Marathon Racing Shoes: A Randomized Crossover Study by Hoogkamer, W., Kipp, S., & Kram, R. (2018). *Sports Medicine*, *49*(1), 133–143.

Figure 10:

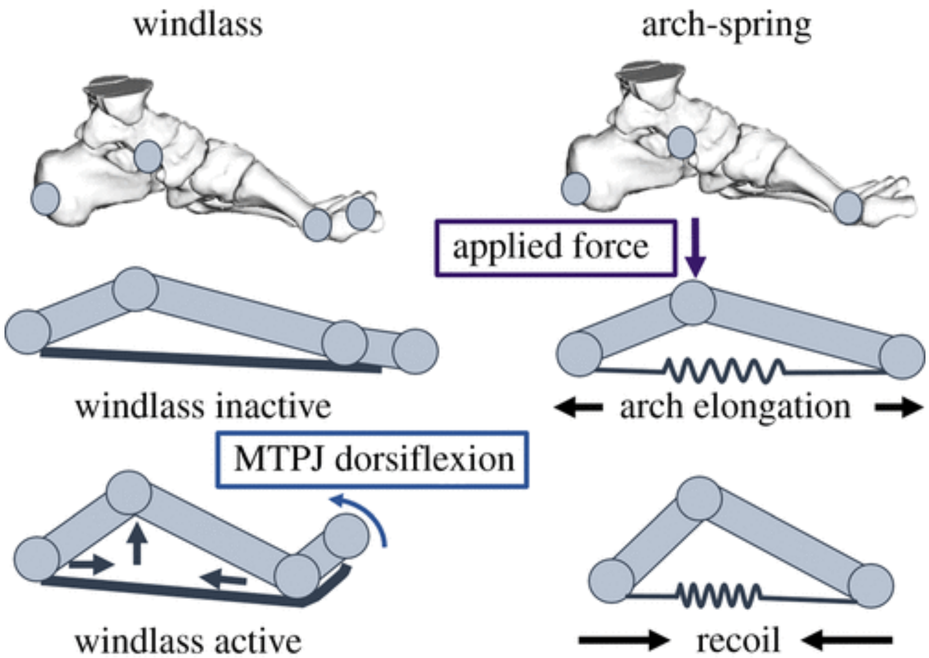
Results of Kluitenberg et al. empirical study on vGRF in running



Note: Results for vGRF during running, test was performed on a treadmill. Adapted from Comparison of vertical ground reaction forces during overground and treadmill running. A validation study by Kluitenberg, B., Bredeweg, S. W., Zijlstra, S., Zijlstra, W., & Buist, I. (2012). *BMC Musculoskeletal Disorders*, *13*(1).

Figure 11:

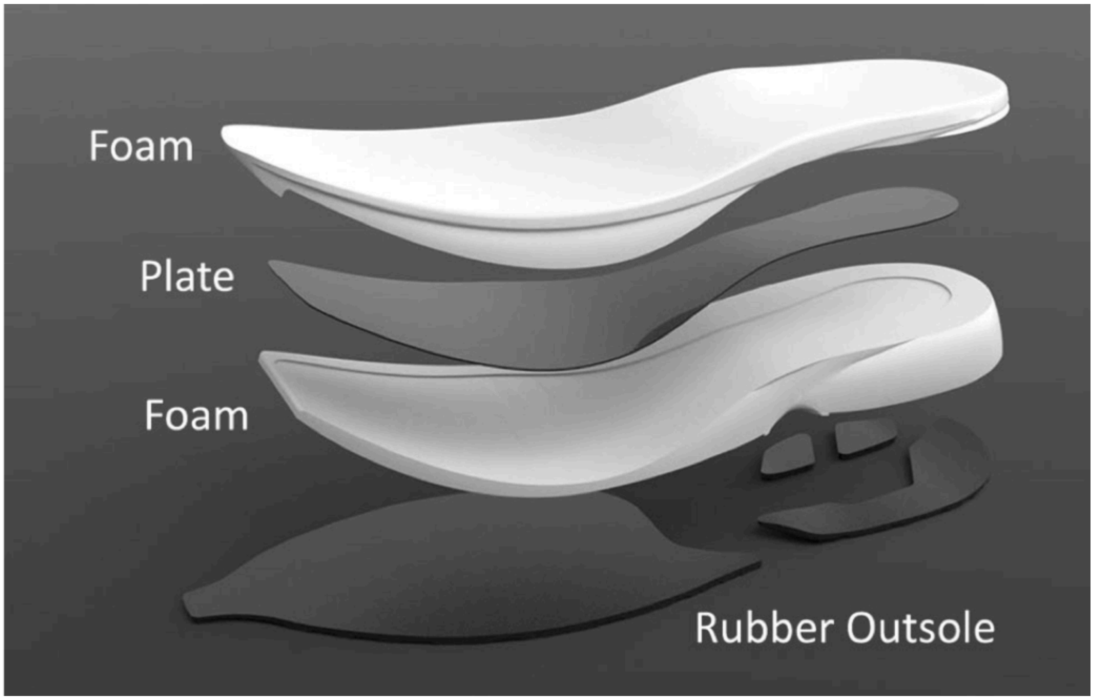
Spring mechanics of the foot diagram



Note: Windlass and Spring Arch mechanism displaying how the foot can act as a spring. Adapted from Influence of the windlass mechanism on arch-spring mechanics during dynamic foot arch deformation by Welte, L., Kelly, L. A., Lichtwark, G. A., & Rainbow, M. J. (2018). *Journal of The Royal Society Interface*, *15*(145)

Figure 12:

Design process for carbon fiber plate in midsole



Note: Carbon fiber plate is designed to go between the foam layers of the midsole in prototype shoe. Adapted from "The Biomechanics of Competitive Male Runners in Three Marathon Racing Shoes: A Randomized Crossover Study by Hoogkamer, W., Kipp, S., & Kram, R. (2018). *Sports Medicine*, *49*(1), 133–143.